# Handling Sequences of Belief Change in a Multi-agent Context

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**Abstract.** This paper focuses on the features of belief change in a multi-agent context. Agents have to consider information received from the other agents and the ability of how to change beliefs after receiving a message is a prerequisite for many multi-agents problems. The agents are embedded in an environment where they are communicating and they have to prevent potential internal conflicts in their beliefs. We study the belief change operators in that context. Our approach is to consider that agents' belief state is a set of pairs ⟨belief, origin of the belief⟩ combined with a preference relation over the agents embedded in the multi-agent system. The belief revision procedure for handling received messages is a syntactic approach which aims at selecting the minimal subsets of the belief base in conflict with the received information and, according to the reliability of the sources of the conflicting beliefs, removing the less reliable beliefs in order to handle the received message. The proposed belief change operators ensure that an agent makes as few changes as possible but also handle iterated belief changes by preserving the order on the beliefs.

## 1 Introduction

It is quite common to characterize intelligent agents in cognitive terms such as the well known belief, desire intention mental attitudes [16]. In that context, belief change is a key problem for agents. In a multi-agent system, agents communicate with each other in order to solve a problem such as building a plan or negotiating a price. When a first agent communicates with a second agent, the first one intends to change the mental state of the second one.
In this paper we focus on how an agent should change its beliefs when it receives new information from the other agents, i.e. how its beliefs should look like after interpreting the received message [15]. We focus here on multi agent systems that exchange messages about a world that do not change. In that context, belief change has to be considered as belief revision. In order to know if an agent has to adopt the received messages, it considers the reliability of the sender. Next, in order to define what beliefs should be dropped to get a new consistent belief base, it drops the beliefs issued from the less reliable agents.

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This criterion is not enough for specifying what beliefs should be dropped since agents may believe statements issued from agents which could be equally reliable. Thus, a second criterion is introduced in order to uniquely define what beliefs the agent should drop. We show that our belief change operators respect most of the AGM postulates.
In our proposal, when the sender of the message is not enough reliable, the receiver agent do not drop the message, it keeps it. As the communicated statement becomes consistent with the current beliefs of the receiver agent after some change actions, this agent will integrate this piece of information in its current belief set. This approach enables agents to consider as much as possible of the received messages. At the same time, agents maintain a set of statements that should not be believed: these statements, called disbeliefs, are justified by messages describing what should not be believed.

The account of belief change we advocate here handles iterated belief change. This is a key feature when we consider autonomous agents, they should be able to take into account a sequence of messages and thus a sequence of belief change actions.

The paper is organized as follows: Section 2 briefly reviews the AGM approach. Section 3 presents the formal definitions for describing an agent's belief state, messages and reliability level of beliefs. In section 4 and 5, we present constructive definitions for the change functions. Section 6 presents the related works and section 7 concludes the paper.

## 2 Preliminaries

The AGM postulates [1] describe how the beliefs of an agent should change according to the main following principles: minimal change and syntax-independence. Let $K$ be a belief set, i.e. a set of propositional formulas, such that $K$ is closed under logical consequence. $K^+_\phi$ is the deductive closure of $K \cup \{\varphi\}$ and it represents the *expansion* of $K$ by $\varphi$. $K^*_\varphi$ represents the *revision* of $K$ by $\varphi$. Let $K_\perp$ be the inconsistent belief set. According to [9], a revision function should satisfy the following postulates:

**(R1)** $K^*_\varphi$ is a belief set.
**(R2)** $\varphi \in K^*_\varphi$.
**(R3)** $K^*_\varphi \subseteq K^+_\varphi$.
**(R4)** If $\varphi \notin K$ then $K^+_\varphi \subseteq K^*_\varphi$.
**(R5)** $K^*_\varphi = K_\perp$ iff $\vdash \neg\varphi$.
**(R6)** If $\vdash \varphi \leftrightarrow \psi$ then $K^*_\varphi = K^*_\psi$.
**(R7)** $K^*_{\varphi\wedge\psi} \subseteq (K^*_\varphi)^+_\psi$.
**(R8)** If $\neg\psi \notin K^*_\varphi$ then $(K^*_\varphi)^+_\psi \subseteq K^*_{\varphi\wedge\psi}$.

And the postulates for the contraction function, $K^-_\phi$ are:

**(C1)** $K^-_\phi$ is a belief set.
**(C2)** $K^-_\phi \subseteq K$.
**(C3)** If $\phi \notin K$ then $K^-_\phi = K$.
**(C4)** If $\nvdash \phi$, then $\phi \notin K^-_\phi$.
**(C5)** If $\phi \in K$, then $K \subseteq (K^-_\phi)\phi^+$.
**(C6)** If $\vdash \phi \leftrightarrow \psi$ then $K^-_\phi = K^-_\psi$;
**(C7)** $K^-_\phi \cap K^-_\psi \subseteq K^-_{\phi \wedge \psi}$;
**(C8)** If $\phi \notin K^-_{\phi\wedge\psi}$ then $K^-_{\phi\wedge\psi} \subseteq K^-_\phi$.

## 3 Agent Beliefs

We assume that beliefs are expressed in a propositional language $\mathcal{L}$. Changes in a belief set are caused by communication only. We assume throughout the paper that the external world is *static*; handling changes caused by "physical" actions would require the integration of belief update to our formalism, which we leave for further work. Thus, we are considering cases such as diagnosis. We assume that messages are sent point-to-point. In order to identify the sender of messages we introduce a set of agent id: let $A = \{a_1 \cdots a_n\}$ be this set.

### 3.1 Describing Messages

In our context, an agent may send two kinds of messages to the other agents: an agent $a$ informs an agent $b$ that $\phi$ holds or an agent $a$ informs an agent $b$ that $\phi$ does not hold. These messages may occur after a request sending by $b$ to $a$ and asking if $\phi$ holds or not. In more formal terms, we get:

**Definition 1 (Message).** *A message $M$ is defined as a tuple of receiver $r$, sender $s$, content $\phi$, status $st$. The receiver and the sender are agent ids, the content is an $\mathcal{L}$-formula and the status is one of the two possible status:* $\{\texttt{Hold}, \texttt{NotHold}\}$. *Self addressed messages are not allowed, i.e. $M = \langle r, s, \phi, st\rangle$ s.t. $s \neq r$ and $\phi \nvdash \bot$. Let $\mathcal{M}$ be the set of all possible messages.*

Remarks $\texttt{NotHold}\phi$ is not equivalent to $\texttt{Hold}\neg\phi$. We do not consider how agents acquire information. The agents interact with the others: they send and receive messages. At each moment the agents may receive a message and they will change their epistemic state.

**Definition 2 (Sequence of messages).** *A sequence of messages $\sigma$ is a function which associates moments in time and messages. Moments in time are represented by the set of integers:* $\sigma : \mathbb{N} \to \mathcal{M}$

### 3.2 Describing Agent Beliefs

The key idea is to represent the "belief state" of an agent as three sets:

- a set of labelled statements representing the current beliefs, i.e. statements are indexing with their corresponding message number (their origins);
- a set of potential beliefs: messages received by the agent which could not be handled since they are in conflict with its current beliefs. Since the current beliefs change with respect to the received messages, some potential beliefs will be consistent with the new current beliefs and thus will be considered as current beliefs in future states.
- a set of "disbeliefs" representing statements that should not be believed by the agent. The set of disbeliefs changes with respect to the flow of messages about statements that do not hold.

To represent beliefs of an agent, we define a signed belief as a pair ⟨statement, origin of the statement⟩ (the received message):

**Definition 3 (Signed belief).** *A signed belief is a pair* $\langle \phi, i \rangle$ *where* $\phi$ *is a* $\mathcal{L}$*-formula and* $i \in \mathbb{N}$ *s.t.* $(\exists r, s, st)(\sigma(i) = \langle r, s, \phi, st \rangle)$ *. Let* $\mathcal{S}$ *be the set of signed beliefs and Let* $\mathcal{SB} = 2^{\mathcal{S}}$ *be the set of all sets of signed beliefs.*

*Example 1.* Let $a_1$ and $a_2$ be two agents and a message $\sigma(1) = \langle a_1, a_2, \neg\varphi, \texttt{Hold} \rangle$. The pair $\langle \neg\varphi, 1 \rangle$ is the associated signed belief.

Based on the set of signed beliefs, we can define what statements could be inferred by an agent:

**Definition 4 (Beliefs set and beliefs base).** *Let Bel be a function which maps a signed beliefs set s to a set of* $\mathcal{L}$*-formulas:* $Bel(s) = \{\psi | \bigwedge_{\langle \phi, i \rangle \in s} \phi \vdash \psi\}$. $Bel(s)$ *represents the* beliefs set *associated to a set s of signed beliefs. Let Base be a function which maps a signed beliefs set s to a set s of* $\mathcal{L}$*-formulas:* $Base(s) = \bigcup_{\langle \phi, i \rangle \in s} \{\phi\}$. $Base(s)$ *is the* beliefs base *associated to s.*

*Example 2.* Let $s = \{\langle \neg\varphi, 1 \rangle, \langle \neg\varphi \rightarrow \varphi', 2 \rangle\}$ be a signed beliefs set. The beliefs set associated to $s$ is: $Bel(s) = Cn(\{\neg\varphi, \neg\varphi \rightarrow \varphi', \varphi'\})$ where $Cn$ is the deductive closure operation; and the beliefs base is equal to $Base(s) = \{\neg\varphi, \neg\varphi \rightarrow \varphi'\}$.

From a set of signed beliefs, we consider the minimal subsets entailing a specific conclusion. Let $\phi$ be a formula and $s$ a set of signed beliefs. Let *support* be a function returning the set of minimal subsets of $s$ entailing $\phi$.

$$support(s, \phi) = \{s' | s' \subseteq s, Bel(s') \vdash \phi \text{ and } \forall s'' \subset s' (s'' \nvdash \phi)\}$$

In order to describe what is believed by an agent, we introduce the notion of epistemic state. An epistemic state describes what is "currently" believed by the agent, what could be potentially believed and what should not be believed. Let us stress that our definition of epistemic states should not be confused with the epistemic states defined by [4,11].

**Definition 5 (Epistemic state).** *An epistemic state is a structure:* $\langle CB, PB, DB \rangle$ *where* $CB \in \mathcal{SB}$*,* $PB \in \mathcal{SB}$ *and* $DB \in \mathcal{SB}$*. The first one represents the current belief of the agent, the second one represents potential belief and the third one represents disbeliefs such that:*

- $CB \in \mathcal{SB}$, $\forall \langle \phi, i \rangle \in CB$ *s.t.* $\sigma(i) = \langle r, s, \phi, \texttt{Hold} \rangle$ *and* $Bel(CB) \nvdash \bot$;
- $(\forall \langle \phi, i \rangle \in PB)(Bel(CB) \wedge \phi \vdash \bot$ *or* $(\exists \langle \phi', i' \rangle \in DB)(Bel(CB) \wedge \phi \vdash \phi'))$;
- $(\forall \langle \phi, i \rangle \in DB)(\sigma(i) = \langle r, s, \phi, \texttt{NotHold} \rangle$ *and* $Bel(CB) \nvdash \phi)$.

According to this definition, (i) potential beliefs are signed beliefs in conflict with the current beliefs and (ii) the belief set associated to the current beliefs do not entail any of the disbeliefs.

*Example 3.* Let $E = \langle \{\langle \neg\varphi, 1\rangle, \langle \neg\varphi \rightarrow \varphi', 2\rangle\}, \{\langle \neg\varphi', 3\rangle, \langle \neg\varphi' \rightarrow \varphi'', 5\rangle\}, \{\langle \varphi'', 4\rangle\}\rangle$ be an epistemic state; i.e. $CB = \{\{\langle \neg\varphi, 1\rangle, \langle \neg\varphi \rightarrow \varphi', 2\rangle\}\}$, $PB = \{\{\langle \neg\varphi', 3\rangle, \langle \neg\varphi' \rightarrow \varphi'', 5\rangle\}\}$ and $DB = \{\{\langle \varphi'', 4\rangle\}\}$ . The signed belief $\langle \neg\varphi', 3\rangle$ belongs to $PB$ since $\neg\varphi'$ contradicts $Bel(CB)$. The signed belief $\langle \neg\varphi' \rightarrow \varphi'', 3\rangle$ also belongs to $PB$ because if it were a member of $CB$ then the disbelief $\langle \varphi'', 4\rangle$ would have been violated. Thus this epistemic is well-defined.

In order to define to what set of signed beliefs should belong a signed belief, the agents consider a procedure for changing their epistemic states.

## 4 Epistemic State Change

In order to handle the messages that agents exchange, each of them needs a specific procedure for handling belief change. The common base of each procedure is an order over the set of agents $\leqslant$: each agent considers their own most reliable sources. Agents that could not be distinguished are considered in an equal way (and thus entailing a total preorder). Since $\leqslant$ is a total preorder, $A$ may be partitioned in $\leqslant$-equivalence classes of equally reliable agents. These classes are themselves totally ordered. The second element is a criterion defining how potential beliefs issued from equally reliable agents should be considered. In this paper we focus on two possible criteria: `NEW` (priority to the newest messages) and `OLD` (priority to the oldest ones). In order words, this criterion combined with $\leqslant$ induces a linear order over the messages received by an agent. Let us mention that there is no meta-criterion for selecting the priority criterion. In fact, any kind of criterion that can be combined with $\leqslant$ so that it induces a linear order over the messages is valid.

**Definition 6 (Agent revision program).** *Let $p$ be an agent revision program defined as a structure: $\langle \leqslant, choice \rangle$. $\leqslant$ is a total preorder over $A$ and choice is a constant belonging* {`NEW`, `OLD`}. *Let $\mathcal{P}$ be the set of agent revision programs.*

Writing $a < b$ means that $b$ is a strictly better source than $a$: $a \leqslant b$ but $b \not\leqslant a$; writing $a = b$ means that $b$ is equal to the source $a$: $a \leqslant b$ and $b \leqslant a$. According to their current epistemic state and their revision program, the agents change their epistemic states as they interact with the other agents. Notice that reliabiliy of agents defined by the revision program is static. We leave for further work this problem: how to rchange the *choice* constant and the pre-order?

### 4.1 Qualifying Beliefs

Each belief of an agent $a$ issued from the set of current beliefs mat be compared to the other signed beliefs according to the order $\leqslant$. This order is refined by considering the constant *choice*.

**Definition 7 ($\prec$).** *Let $p$ be an agent revision program of an agent $r$. Let $a = \langle \phi, i \rangle$ and $b = \langle \psi, j \rangle$ be two signed beliefs s.t. $\sigma(i) = \langle r, s, \phi, st \rangle$ and $\sigma(j) = \langle r, s', \psi, st' \rangle$. $a \prec b$ iff:*

- *if $s < s'$ or*
- *if $s = s'$ and $i <_{\mathbb{N}} j$ and choice is defined in $p$ as* `OLD`*;*
- *if $s = s'$ and $i >_{\mathbb{N}} j$ and choice is defined in $p$ as* `NEW`*.*

Let us notice that $\prec$ is a linear order

## 5 The Change Functions

In this section, we describe the three kinds of belief change: expansion, revision and contraction. Let us consider a set of agents $A$ where their initial epistemic states is empty: $(\forall a \in A) E_a^0 = \langle \emptyset, \emptyset, \emptyset \rangle$, a set of revision programs $prog : A \to \mathcal{P}$ and a sequence of messages $\sigma$. Messages received by the agents entail a revision or contraction action of their epistemic state. Now we describe the three kinds of belief change: expansion, revision and contraction. Let us consider a set of agents $A$ where their initial epistemic states is empty: $(\forall a \in A) E_a^0 = \langle \emptyset, \emptyset, \emptyset \rangle$, a set of revision programs $\mathcal{P}$ and a sequence of messages $\sigma$. Messages received by the agents entail a revision or a contraction action of their epistemic states. Let $n \in \mathbb{N}$ and $\sigma(n) = \langle r, s, \phi, status \rangle$ be a message. Suppose $E_r^n = \langle CB, PB, DB \rangle$ be the epistemic state of the agent $r$ at $n$. The epistemic state of $r$ is recursively defined accordingly to $p$, its revision program, for any message received by $r$ at $n' < n$. Since one message is sending at $n$, only one agent changes its epistemic state.

**Definition 8.** *Let $A$ be a set of agents, $E_a^n$ be the epistemic states of every agent $a$ and $\sigma(n) = \langle r, s, \phi, status \rangle$ be a message.*

$$E_a^{n+1} = \begin{cases} (E_a^n) & \text{if } a \neq r \\ (E_a^n)^*_{\langle \phi, n \rangle} & \text{if } status = \texttt{Hold} \text{ and } a = r \\ (E_a^n)^-_{\langle \phi, n \rangle} & \text{if } status = \texttt{NotHold} \text{ and } a = r \end{cases}$$

Expansion are not directly caused, however we introduce this change action since AGM postulates used it.

### 5.1 Expansion

Let $sb = \langle \phi, i \rangle$ s.t.$\sigma(i) = \langle a, s, \phi, \texttt{Hold} \rangle$ and $E$ the epistemic state of the agent $a$. Classically, expansion are based on the deductive closure of the belief set:

$$E_{sb}^+ = \langle CB^+, PB^+, DB^+ \rangle = \langle CB \cup \{sb\}, PB, DB \rangle$$

Let $K'$ be the current belief set associated to $E^+_{sb}$. By definition $K' = Bel(CB) \cup \{\phi\}$ and thus our expansion operator follows the AGM postulates for this operator [9].

### 5.2 Revision

Agents are autonomous: for each message they receive, they use their own revision program in order to prevent inconsistencies in their beliefs. In other words, the selection mechanism that select signed beliefs to be remove must not be specific to a belief state but must be appropriate for handling multiple belief change. Thus, the agent revision program is able to handle iterated belief change [4,11]. In order to handle this, agents have to determine what beliefs should be dropped. Here we are considering beliefs in their syntactic form, i.e. the messages, and thus revision will be based on this approach [8,13,3]. The procedure for revising epistemic states is based on the safe revision function [2]

Let $n$ be a moment, $M = \sigma(n) = \langle r, s, \phi, st\rangle$ be a message such that $st = \texttt{Hold}$, and $E^n = \langle CB, PB, DB\rangle$ be the epistemic state of the agent $r$ at the moment $n$. The epistemic state is recursively defined accordingly to $prog(r)$ for any message received at a moment $m < n$ by $r$.
In a first stage, the current belief set has to be revised with respect to the set of disbeliefs. Next, in a second stage all the potential beliefs consistent with the new current belief set will be added to the new current belief set and removed from the set of potential beliefs. Finally, the disbeliefs that must be no longer considered are removed.

**First stage: revising $CB$.** Here, we use the degree of each conflicting belief defined by the total preorder $\leqslant$. First, we consider the set of minimal subsets of $\mathcal{L}$-statements $\Gamma$ issued from $CB$ entailing $\neg\phi$: $\Gamma = support(CB, \neg\phi)$. In a second stage we remove the less credible signed belief associated to each minimal subset $\gamma \in \Gamma$. The order $\prec$ ensures us that there exists a unique minimal element in $\gamma$ ($min(\gamma)$). In order to define what beliefs have to be be dropped, we employ a complete pre-order with maximal elements on the entire set $\Gamma$ [13]. We say that a set $\gamma_1$ is preferred to a set $\gamma_2$, $\gamma_2 \ll \gamma_1$, if the maximal element of $\gamma_1$ is preferred to the maximal element of $\gamma_2$; $\gamma_1$ is equally preferred to $\gamma_2$, $\gamma_2 = \gamma_1$, if the maximal element of $\gamma_1$ is equal to the maximal element of $\gamma_2$. Note that such maximal elements of $\gamma$ always exist. Let $esr$ be a safe revision function based on $\ll$.

We remove minimal elements in every set $\gamma$, one by one, in an iterative way. Let $CB^0 = CB$, $\Gamma^0 = \Gamma$ and $\mathsf{min}(\Gamma^0)$ the set of minimal elements of $\Gamma^0$ with respect to $\ll$. Let $a$ be the signed belief being the minimal element belonging to the union of the elements of $\mathsf{min}(\Gamma^0)$ (w.r.t. $\prec$). The revision function, $esr$, for revising the current signed beliefs set removes the least preferred element to each minimal subset entailing $\neg\phi$.

**Definition 9 ($esr$).** *The revision of the current signed beliefs of an epistemic state $E$ by a signed belief $sb = \langle \phi, n\rangle$, s.t. $\sigma(n) = \langle r, s, \phi, \texttt{Hold}\rangle$ is defined as $esr(E, sb) = CB^{|\Gamma|} \cup \{sb\}$ where $|\Gamma|$ represents the number of elements of $\Gamma$ and such that:*

- *let $a = \langle \psi, i\rangle$ s.t.:*
  1. *$\exists \gamma \in \mathsf{min}(\Gamma^i)$ s.t. $a \in \gamma$,*
  2. *$\forall \gamma \in \mathsf{min}(\Gamma^i), \nexists \langle \psi', i'\rangle \in \gamma$ s.t. $\langle \psi', i'\rangle \prec \langle \psi, i\rangle$.*
- *$CB^{i+1} = CB^i - \{a\}$ and*
- *$\Gamma^{i+1} = \Gamma^i - \{\gamma | \gamma \in \mathsf{min}(\Gamma^i)$ and $a \in \gamma\}$.*

**Theorem 1.** *Let $r$ be an agent, $p$ be its agent revision program, $E^n = \langle CB, PB, DB\rangle$ its epistemic state and a message $\sigma(n) = \langle r, s, \phi, \texttt{Hold}\rangle$. The revision function $esr$ determined by $E^n$ and $p$ satisfies postulates (R1) through (R5), (R7) and (R8) when $esr(E^n, \langle \phi, n\rangle) = CB^{|\Gamma|} \cup \{\langle \phi, n\rangle\}$.*

(R6) is not satisfied since $esr$ is a syntax-based function[1].

**Second stage: adding/removing potential beliefs and disbeliefs.** After revising its current set of signed beliefs, the agent has to consider all the potential beliefs. These beliefs must not entail (i) an inconsistency in the current beliefs and (ii) any of the disbeliefs. For this, we consider the most preferred potential beliefs.

**Definition 10 ($\Pi$ and $\Delta$).** *The set $\Pi$ of potential beliefs to add to the new current belief set and the set $\Delta$ of potential disbeliefs that have to be considered as disbeliefs are defined as follows:*

- *$PB^0 = PB$, $\Pi^0 = \emptyset$, $\Delta^0 = \emptyset$;*
- *let $m = \langle \psi, j\rangle = max(PB^i)$ with respect to $\prec$ defined over the set $PB^i$;*
- *if $\sigma(j) = \langle r, s', \psi, \texttt{Hold}\rangle$ (i.e. the status is* `Hold`*) and*
  1. *the belief set $Bel(esr(E, sb) \cup \Pi^i \cup \{m\}) \nvdash \bot$;*
  2. *$\forall \langle \psi', j'\rangle \in DB \cup \Delta^i$ if $\forall \gamma \subseteq esr(E, sb) \cup \Pi^i \cup \{m\}$ s.t. $\bigwedge_{\langle \psi'', l\rangle \in \gamma} \psi'' \vdash \psi'$, $\gamma$ is minimal for $\phi'$ and $m \in \gamma$ then $\{\langle \psi', j'\rangle\} \ll \gamma$.*

  *then $\Pi^{i+1} = \Pi^i \cup \{m\}$ and $\Delta^{i+1} = \Delta^i$;*
- *if $\sigma(j) = \langle r, s', \psi, \texttt{NotHold}\rangle$ and $Bel(esr(E, sb) \cup \Pi^i) \nvdash \psi$ then $\Delta^{i+1} = \Delta^i \cup \{m\}$ and $\Pi^{i+1} = \Pi^i$;*
- *$PB^{i+1} = PB^i$ - $\{m\}$.*

*The sets $\Pi, \Delta$ are equal to $\Pi^{|PB|}$ and $\Delta^{|PB|}$.*

The sets $\Pi$ and $\Delta$ are removed from the potential beliefs. Finally, the disbeliefs that should be no longer considered have to be removed from $DB$ and transferred into $PB$. Let $\bar{\Delta}$ be the set of this disbeliefs:

$$\bar{\Delta} = \{\langle \psi, j\rangle | \sigma(j) = \langle r, s', \psi, \texttt{NotHold}\rangle \text{ and } Bel(esr(E, \langle \phi, n\rangle) \cup \Pi) \vdash \psi \text{ and } (\exists \gamma \in support(esr(E, \langle \phi, n\rangle, \psi)) \langle \psi, j\rangle \prec min(\gamma)\}$$

[1] Postulates are reformulated w.r.t. our definitions; for instance (R1): $esr(E^n, \langle \phi, n\rangle)$ is a set of signed beliefs (i.e. $Bel(esr(E^n, \langle \phi, n\rangle))$ is a belief set).

When we revise the epistemic state of the agent, $E^*_{\langle\phi,n\rangle} = \langle CB^*, PB^*, DB^*\rangle$, we distinguish two cases: the degree of $\phi$, i.e. $s$ is lower than the highest degree of a conflicting belief of $r$ and the case where the sending agent has to be trusted. If the resulting set $CB^{|\Gamma|} \cup \{sb\}$ (cf. definition 9) entails that a disbelief $\psi$ holds where the degree of this disbelief is better than the degree of the new belief then the revision is canceled. The revision action is also canceled when $\phi$ is weaker than the minimal statement supporting $\neg\phi$.

**Definition 11 (∗).** *Let $E^*_{\langle\phi,n\rangle} = \langle CB^*, PB^*, DB^*\rangle$ be the epistemic state of the agent $r$ at the moment $n+1$:*

- *if $(\exists\langle\psi,k\rangle \in DB$ s.t. $esr(E,\langle\phi,n\rangle) \vdash \psi$ and $\langle\phi,n\rangle \prec \langle\psi,k\rangle)$ or $(Bel(CB) \vdash \neg\phi$ and $\exists\gamma \in support(CB,\neg\phi)$ s.t. $\langle\phi,n\rangle \prec min(\gamma))$ then $E^*_{\langle\phi,n\rangle} = \langle CB, PB \cup \{\langle\phi,n\rangle\}, DB\rangle$*
- *else $E^*_{\langle\phi,n\rangle} = \langle esr(E,\langle\phi,n\rangle) \cup \Pi, PB - \Pi - \Delta \cup \bar{\Delta}, DB \cup \Delta - \bar{\Delta}\rangle$*

### 5.3 Contraction

In this section, we briefly describe the contraction operator $-$. This action is very close to the revision operator $*$. The contraction operator is also a two stage operator: at first the current beliefs are contracted if the sender of the message is reliable and secondly, the potential beliefs that are consistent with the resulted current beliefs are added to them.

Let $n$ be a moment, $M = \sigma(n) = \langle r, s, \phi, st\rangle$ be a message such that $st = \texttt{NotHold}$, and $E^n = \langle CB, PB, DB\rangle$ be the epistemic state of the agent $r$ at the moment $n$.

**First stage: contracting $CB$.** The process for contracting the current beliefs is close to the process for revising $CB$. We characterize all the minimal subsets entailing the statement which have to be removed. Let $\Gamma = support(CB,\phi)$. Next, as for the function $esr$, we iteratively remove the less reliable beliefs from each set $\gamma \in \Gamma$. Definitions for $\ll$ and $\mathsf{min}(\Gamma)$ are unchanged.

**Definition 12 ($esc$).** *The contraction of the current signed beliefs of an epistemic state $E$ by a signed belief $sb = \langle\phi,n\rangle$ is defined as $esc(E,sb) = CB^{|\Gamma|}$ where $CB^{|\Gamma|}$ is defined as follows:*

- *Let $a = \langle\psi,i\rangle$ be a signed belief s.t.:*
  1. *$\exists\gamma \in \mathsf{min}(\Gamma^i), a \in \gamma$,*
  2. *$\forall\gamma \in \mathsf{min}(\Gamma^i), \nexists\langle\psi',i'\rangle \in \gamma$ s.t. $\langle\psi',i'\rangle \prec \langle\psi,i\rangle$.*
- *$CB^{i+1} = CB^i - \{a\}$;*
- *$\Gamma^{i+1} = \Gamma^i - min(\Gamma^i)$;*

**Theorem 2.** *Let $r$ be an agent, $p$ be its agent revision program, $E^n = \langle CB, PB, DB\rangle$ its epistemic state and a message $\sigma(n) = \langle r, s, \phi, st\rangle$. The contraction function esc determined by $E^n$ and $\langle\phi,n\rangle$ satisfies postulates (C1) through (C5), (C7) and (C8) when $esc(E,\langle\phi,n\rangle) = CB^{|\Gamma|}$.*

(C6) is not satisfied since our approach is a syntax-based approach.

**Second stage: adding/removing potential beliefs and disbeliefs.** After contracting the current set of signed beliefs of the agent, the agent $r$ has to consider all the potential beliefs since these beliefs may be consistent with the set $esc(E, s)$. Definitions for $\Pi$ and $\Delta$ are unchanged (the set $\bar{\Delta}$ is not concerned by the contraction operation).

**Definition 13 (-).** *Let* $E^-_{\langle\phi,n\rangle} = \langle CB^-, PB^-, DB^-\rangle$ *be the epistemic state of the agent* $r$ *at the moment* $n+1$ *s.t.:*

- *if* $Bel(CB) \vdash \phi$ *and* $\exists\gamma \in support(CB, \phi)$ *s.t.* $\langle\phi, n\rangle \prec min(\gamma)$ *then* $E^-_{\langle\phi,n\rangle} = \langle CB, PB \cup \{\langle\phi, n\rangle\}, DB\rangle$
- *else* $E^-_{\langle\phi,n\rangle} = \langle esc(E, \langle\phi, n\rangle) \cup \Pi, PB - \Pi - \Delta, DB \cup \Delta \cup \{\langle\phi, n\rangle\}\rangle$

Current beliefs which have been removed by the contraction function *esc* are adding to the potential beliefs and if the received message do not entail a contraction then it is considered as a "potential disbelief".

### 5.4 An Example

Let us consider three agents $a_1$, $a_2$, $a_3$. The revision program of the agent $a_1$ is: $prog(a_1) = \langle\{(a_1 < a_2, a_2 < a_3)\}, \texttt{OLD}\rangle$ and the initial state is $E^0_{a_1} = \langle\emptyset, \emptyset, \emptyset\rangle$. Let us consider the following sequence of 5 messages: 0, 1, 2, 3 and 4

$$\sigma = [\langle a_1, a_2, \phi, \texttt{Hold}\rangle, \langle a_1, a_3, \phi \to \psi, \texttt{Hold}\rangle, \langle a_1, a_3, \psi, \texttt{Hold}\rangle, \langle a_1, a_3, \psi, \texttt{NotHold}\rangle, \langle a_1, a_2, \psi, \texttt{Hold}\rangle]$$

According to the revision program and the change operators, we get the following epistemic states for the agent $a_1$:

$$E^1 = \langle\{\langle\phi, 0\rangle\}, \emptyset, \emptyset\rangle$$
$$E^2 = \langle\{\langle\phi, 0\rangle, \langle\phi \to \psi, 1\rangle\}, \emptyset, \emptyset\rangle$$
$$E^3 = \langle\{\langle\phi, 0\rangle, \langle\phi \to \psi, 1\rangle, \langle\psi, 2\rangle\}, \emptyset, \emptyset\rangle$$
$$E^4 = \langle\{\langle\phi \to \psi, 1\rangle\}, \{\langle\phi, 0\rangle, \langle\psi, 2\rangle\}, \{\langle\psi, 3\rangle\}\rangle$$
$$E^5 = \langle\{\langle\phi \to \psi, 1\rangle\}, \{\langle\phi, 0\rangle, \langle\psi, 2\rangle, \langle\psi, 4\rangle\}, \{\langle\psi, 3\rangle\}\rangle$$

Messages 0, 1, 2 and 4 entail a revision operation while message 3 entails a contraction operation. Let us notice that the last message ($a_2$ informs $a_1$ that $\psi$ holds) do not imply that $\psi$ is believed by $a_1$. This is due to disbelief $\psi$ which is stronger (since it has been send by $a_3$ and $a_3$ is better than $a_2$). The beliefs set associated to the current beliefs set of $E^5$ is $K = \{\alpha | \phi \to \psi \vdash \alpha\}$.

## 6 Related Works

Several works have already been done in the multi-agent revision area. A. Dragoni and P. Giorgini have presented a similar framework [5,6,7]. They proposed

to consider belief revision in a multi-agent system by considering the origin of information. Based on the reliability of the incoming statements, agents can also reject new information. Agents can also reconsider all the previous messages and more specifically discarded beliefs can be reinstated. The main difference with our work is that they do not consider disbeliefs in an explicit way and thus they do not take into account contraction operations.
W. Liu and M.A. Williams propose in [12] a framework for handling belief revision. After distinguishing private and public knowledge, the authors describe a belief revision engine aiming at revising beliefs sharing by several agents. They propose this framework since agents should be able to consider common beliefs. We have made a different choice since we consider beliefs as private. However, our definition of beliefs can handle semi-public beliefs since every statement is qualified with its origin. Thus agents can "establish" some common beliefs. At the opposite, in [12], agents can only consider positive beliefs and they do not take into account at each stage the sequences of previously received messages.
Concerning the iterated belief revision area, our work is loosely connected to [4] and [11] since we consider that a revision action may fail. Thus, the proposed postulates for qualifying iterated revision processes are not satisfied by our framework.

## 7 Conclusion

In this paper, we have presented belief change operators for handling iterated belief change. Our work focus on belief change when agents exchange information about a static world such as diagnosis. The first characteristic of our operators is to preserve an order over the beliefs. Because this information is retained, any agent may handle any sequence of messages. The second characteristic of our framework is to keep all the received messages and to reconsider them for taking care of what the agents should not believe and, according to their belief changes, how to reconsider previous messages (represented as potential beliefs).
Our approach does have some limitations. In this paper, we only look at cases where belief change concerns a static world. We only consider communication actions. If we consider more general actions, instead of beliefs changing only through messages, changes could result from sensing actions. In a short term, we would like to generalize our framework in order to handle these limitations, i.e. update rather than revision. Another direction of research for handling changes is to consider different policies for adopting/rejecting a message: effect of convergence, feed-back for getting a fine-grain reliability of information... In a more longer term, our aim is to enable agents to reason about their current beliefs and past beliefs so that they can initiate messages for acquiring new information. Another direction is to reconsider treatments of the semantics of the modal operators for describing beliefs in BDI models: a semantics based on the messages and epistemic states will be useful for describing not only beliefs and disbeliefs [14] but also beliefs change [10] and how agents may be aware about their beliefs and disbeliefs. We are currently investigating all these topics.

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